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NOTE ON THE SOLUTION OF PROB. 363, BY PROF. W. W. JOHNSON.—The following proof of Prob. 363 shows that the theorem is true for all convex ovals.

Let O be the fixed point and AB the chord, the tangent at B being parallel to OA; and complete the parallelogram $AOBT$. As B travels about the oval the tangent BT is at every instant rotating about B; Hence, since BT is equal and parallel to OA it generates an area equal to that generated by OA, that is, an area equal to the given oval. Thus the area of the locus of T is double that of the oval. Now the middle point of AB is also the middle point of OT, hence its locus is similar to that of T, and its area is one fourth the area of the locus of T or one half the area of the given oval.

ANOTHER SOLUTION OF PROB. 365, BY PROF. ASAPH HALL.—It is plain that the indetermination will occur only for very small values of θ. Put therefore $\cos\theta = 1 - \frac{1}{2}\theta^2$, and, neglecting higher powers of θ, we shall have,

$$\int_0^\theta \frac{\sqrt{(1-c)}.d\theta}{1-c+\frac{1}{2}nc.\theta^2} = \sqrt{\frac{2}{nc}}\,.\,\tan^{-1}\frac{\theta.\sqrt{(nc)}}{\sqrt{[2(1-c)]}};$$

$$= \sqrt{\frac{2}{n}}\,.\,\frac{\pi}{2} = \frac{\pi}{\sqrt{(2n)}},$$

when $c = 1$. This is the value required, since the upper limit may be changed from θ to $\frac{1}{2}\pi$.

SOLUTIONS OF PROBLEMS IN NUMBER SIX, VOL. VIII.

SOLUTIONS of problems in No. 6, Vol. VIII, have been rec'd as follows: From R. J. Adcock, 374; Prof. W. P. Casey, 368, 370, 371; George E. Curtis, 370; Dr. H. Eggers, 368, 370; Prof. A. B. Evans, 371; Prof. E. J. Edmunds, 368, 369, 370; George Eastwood, 372; Prof. E. W. Hyde, 369; W. E. Heal, 369; William Hoover, 372; Prof. J. Scheffer, 368, 369, 370, 372; Prof. E. B. Seitz, 369, 370, 371; Thos. Spencer, 369; R. S. Woodward, 374.

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368. *By Prof. J. Scheffer.*—"In a quadrilateral $ABCD$, the diagonal AC makes with the sides the four angles $CAB = \alpha$, $ACB = \beta$, $ACD = \gamma$, $CAD = \delta$. Find the angles which the other diagonal BD makes with the sides."

SOLUTION BY PROF. J. SCHEFFER, HARRISBURG, PA.

Denoting $\angle BDC$ by θ, and $\angle BDA$ by φ, we have

$$BC : CD = \sin\theta : \sin(\beta+\gamma+\theta),$$
$$CD : AC = \sin\delta : \sin(\delta+\gamma).$$

Multiplying:

$$BC : AC = \sin\delta\sin\theta : \sin(\delta+\gamma)\sin(\beta+\gamma+\theta);$$

but $BC : AC = \sin\alpha : \sin(\alpha+\beta)$; therefore

$$\sin\alpha : \sin(\alpha+\beta) = \sin\delta\sin\theta : \sin(\delta+\gamma)\sin(\beta+\gamma+\theta),$$

whence $\sin\alpha\sin(\delta+\gamma)\sin(\beta+\gamma+\theta) = \sin(\alpha+\beta)\sin\delta\sin\theta$, or

$$\sin\alpha\sin(\delta+\gamma)[\sin(\beta+\gamma)\cos\theta+\cos(\beta+\gamma)\sin\theta]$$
$$=\sin(\alpha+\beta)\sin\delta\sin\theta$$

Dividing by $\sin\theta$, we obtain

$$\cot\theta = \frac{\sin\delta\sin(\alpha+\beta)}{\sin\alpha\sin(\beta+\gamma)\sin(\delta+\gamma)} - \cot(\beta+\gamma).$$

Similarly

$$\cot\varphi = \frac{\sin\gamma\sin(\alpha+\beta)}{\sin\beta\sin(\alpha+\delta)\sin(\delta+\gamma)} - \cot(\alpha+\delta).$$

Substituting the values of θ and φ, as here found, in the equations

$$\angle ABD = \pi-(\alpha+\delta+\varphi),$$
$$\angle CBD = \pi-(\beta+\gamma+\theta),$$

these angles also become known.

[Dr. Eggers' solution of this problem is similar to the foregoing. Prof. Casey assumes that the sides of the quadrilateral are given, and hence concludes that the solution involves only the application of a well known case in trigonometry. Prof. Edmunds obtains, from the figure, four equations involving the four unknown angles with known quantities, by the reduction of which, he assumes, the solution may be effected. His method, however, will not succeed because his equations are not independent.—Ed.]

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369. *By R. J. Adcock.*—"Show that the radius of curvature of an ellipse equals the cube of the radius vector divided by the rectangle of the semi axes; the radius vector being through the centre at right angles to the radius of curvature."

SOLUTION BY PROF. E. B. SEITZ, KIRKSVILLE, MISSOURI.

Let R be the radius of curvature at the point $(a\cos\varphi, b\sin\varphi)$, and r the radius vector from the center perpendicular to R. Then by the usual formula we have $R=\surd(a^2\sin^2\varphi+b^2\cos^2\varphi)^3\div ab$ (1). Since r is parallel to the tangent at $(a\cos\varphi, b\sin\varphi)$, and the radius vector of this point is $\surd(a^2\cos^2\varphi+b^2\sin^2\varphi)$, we have $r=\surd(a^2\sin^2\varphi+b^2\cos^2\varphi)$, (2). From (1) & (2) $R=r^3\div ab$.

370. *By Prof. Edmunds.*—"Divide a right angle into three parts α, β, γ, such that $(\cos\alpha)\div m = (\cos\beta)\div n = (\cos\gamma)\div p$."

SOLUTION BY DR. H. EGGERS, MILWAUKEE, WISCONSIN.

Construct a triangle with m, n, p as sides; then the complements α, β, γ of its three angles are the angles required.

For if a, b, c be the angles of the triangle, we have

$$m : n : p = \sin a : \sin b : \sin c,$$
$$= \cos(\tfrac{1}{2}\pi - a) : \cos(\tfrac{1}{2}\pi - b) : \cos(\tfrac{1}{2}\pi - c).$$

Now $\alpha + \beta + \gamma = (\tfrac{1}{2}\pi - a) + (\tfrac{1}{2}\pi - b) + (\tfrac{1}{2}\pi - c)$

$$= \tfrac{3}{2}\pi - (a + b + c)$$
$$= \tfrac{3}{2}\pi - \pi = \tfrac{1}{2}\pi.$$

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371. *By Prof. E. B. Seitz.*—"ACB is the quadrant of a circle, O the center of its inscribed circle; O_1, O_2, O_3, . . . O_n are the centers of a series of circles, each of which touches the preceding, the arc AB and the radius AC, the circle O_1 touches the circle O; and OH, O_1H_1, O_2H_2, . . . O_nH_n are the perpendiculars on AC, or the radii of the inscribed circles. If $AC = r$, $O_nH_n = x_n$, and $CH_n : O_nH_n = u_n$, prove that

$$u_n = \tfrac{1}{2}(\sqrt{2} + 1)^{2n+1} - \tfrac{1}{2}(\sqrt{2} - 1)^{2n+1},$$

$$x_n = \frac{2r}{2 + (\sqrt{2} + 1)^{2n+1} + (\sqrt{2} - 1)^{2n+1}}."$$

SOLUTION BY PROF A. B. EVENS, LOCKPORT, N. Y.

Let $CH_n = y_n$; then from the geometry of the figure

$$4x_n y_{n-1} = (y_n - y_{n-1})^2, \qquad (1)$$

$$x_n = \frac{1}{2r}(r^2 - y_n^2). \qquad (2)$$

Similarly

$$x_{n-1} = \frac{1}{2r}(r^2 - y_{n-1}^2). \qquad (3)$$

$$\therefore 4x_n x_{n-1} = \frac{1}{r^2}(r^2 - y_n^2)(r^2 - y_{n-1}^2). \qquad (4)$$

From (1) and (4), by elimination and evolution,

$$r^2 - y_n y_{n-1} = (y_n - y_{n-1})r\sqrt{2}. \qquad (5)$$

From (5), by solving for y_n and then for y_{n-1}, we find

$$y_n = r(r + y_{n-1}\sqrt{2})\div(r\sqrt{2} + y_{n-1}), \qquad (6)$$

and

$$y_{n-1} = r(y_n\sqrt{2} - r)\div(r\sqrt{2} - y_n); \qquad (7)$$

$$\therefore \quad y_{n-2} = r(y_{n-1}\sqrt{2} - r)\div(r\sqrt{2} - y_{n-1}). \qquad (8)$$

By eliminating y_n between (2) and (6) we find

$$x_n = \tfrac{1}{2}r(r^2 - y_{n-1}^2) \div (r\sqrt{2} + y_{n-1})^2, \tag{9}$$

and by writing $n-2$ for n in (2) and then eliminating y_{n-2} by aid of (8),

$$x_{n-2} = \tfrac{1}{2}r(r^2 - y_{n-1}^2) \div (r\sqrt{2} - y_{n-1})^2. \tag{10}$$

By aid of (6), (8), (9), and (10) we may write

$$u_n = \frac{y_n}{x_n} = \frac{2}{r^2 - y_{n-1}^2}\left\{ 3ry_{n-1} + (r^2 + y_{n-1}^2)\sqrt{2} \right\}$$

and $$u_{n-2} = \frac{y_{n-2}}{x_{n-2}} = \frac{2}{r^2 - y_{n-1}^2}\left\{ 3ry_{n-1} - (r^2 + y_{n-1}^2)\sqrt{2} \right\};$$

whence $$u_n + u_{n-2} = \frac{12ry_{n-1}}{r^2 - y_{n-1}^2} = 6u_{n-1}.$$

$$\therefore u_n - 6u_{n-1} + u_{n-2} = 0. \tag{11}$$

The solution of (11) by Finite Differences gives

$$u_n = C_1(r_1)^n + C_2(r_2)^n; \tag{12}$$

where $r_1 = (\sqrt{2} + 1)^2$ and $r_2 = (\sqrt{2} - 1)^2$ are the roots of the equation $a^2 - 6a + 1 = 0$, and C_1 and C_2 are constants of integration.

To determine these constants, observe that when $n = 0$ and $n = 1$, $u = 1$ and $u_1 = 7$, and therefore $1 = C_1 + C_2$ and $7 = C_1(\sqrt{2}+1)^2 + C_2(\sqrt{2}-1)^2$; whence $C_1 = \frac{1}{2}(\sqrt{2} + 1)$ and $C_2 = \frac{1}{2}(\sqrt{2} - 1)$. These values of C_1 and C_2 reduce (12) to $u_n = \frac{1}{2}(\sqrt{2}+1)^{2n+1} - \frac{1}{2}(\sqrt{2}-1)^{2n+1}$. (13)

Since $y_n \div x_n = \sqrt{(r^2 - 2rx_n)} \div x_n = u_n$, we readily find from (13),

$$x_n = \frac{2r}{2 + (\sqrt{2}+1)^{2n+1} + (\sqrt{2}-1)^{2n+1}}.$$

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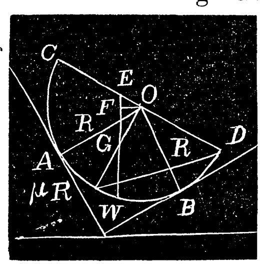
372. *By William Hoover, A. M.*—"A hemisphere, radius r, is resting with its convex surface on two planes, one perfectly smooth and inclined to the horizon at an angle α, the other being inclined at an angle β; if m be the coefficient of friction between the latter and the hemisphere, what is the position for rest?"

SOLUTION BY GEORGE EASTWOOD, SAXONVILLE, MASS.

Let θ represent the angle the base makes with the horizon when the hemisphere is at rest; G, its centre of grav. and EGW a vertical line through G.



We have for data, $\theta = EOF = EGO$; R, μR, and R' for forces, and α and β for inclinations of planes; also $OG = \frac{3}{8}r$.

Take vertical and horizontal components of R, μR, R'; and moments about centre O. Then

$$R\cos\alpha + \mu R\sin\alpha + R'\cos\beta = W, \tag{1}$$
$$R\sin\alpha + \mu R\cos\alpha - R'\sin\beta = 0, \tag{2}$$
$$r.\mu R = FO.W = \tfrac{3}{8}r.W\sin\theta. \tag{3}$$

Multiply (1) by $\sin\beta$, and (2) by $\cos\beta$, and add: then

$$R\sin(\alpha+\beta)+\mu R\cos(\alpha-\beta) = W\sin\beta.$$

$$\therefore R = \frac{W\sin\beta}{\sin(\alpha+\beta)+\mu\cos(\alpha-\beta)}. \tag{4}$$

Substituting this value of R in (3), we have

$$\sin\theta = \frac{8\mu\sin\beta}{3\sin(\alpha+\beta)+3\mu\cos(\alpha-\beta)}.$$

[Solved in a similar manner by the proposer, and by Prof. Scheffer.]

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373. No solution received.

374. *By R. S. Woodward.*—"Prove 1st, that the probable error of any tabular value in a table of logarithms, trigonometric functions, etc., is 0.25 of a unit of the last decimal place, supposing this place correct to the nearest unit; 2nd, that the average of the squares of probable errors of interpolated values depending on first differences only is $\frac{2}{3}(0.25)^2$."

SOLUTION BY THE PROPOSER.

The actual errors of tabular values are confined within the limit $+0.5$ and -0.5 of a unit of the last place. All errors between these limits are equally probable. Hence the probable error of any tabular value is one-half the maximum error, or ± 0.25.

Let v and v' be two consecutive tabular values, and x an interpolated value $\frac{1}{10}t$ from v. Then

$$x = v + \tfrac{1}{10}t(v'-v) = v(1-\tfrac{1}{10}t) + v'\tfrac{1}{10}t.$$

The square of the probable error of x is

$$(\text{p. e. } x)^2 = (0.25)^2\left\{\left(1-\frac{t}{10}\right)^2+\frac{t^2}{100}\right\}$$

$$= (0.25)^2\left\{1-2\frac{t}{10}+2\frac{t^2}{100}\right\}. \tag{1}$$

The average of the squares of probable errors given by this formula between the limits $t=0$ and $t=10$ is

$$(0.25)^2\int_0^{10}\left(1-2\frac{t}{10}+2\frac{t^2}{100}\right)dt \div \int_0^{10}dt = \tfrac{2}{3}(0.25)^2.$$

From (1) it appears that the probable error of an interpolated value is always less than that of a tabular value, and that the probable error is least for the interpolated values midway between the two tabulated values.

[R. J. Adcock submits the following remarks on the solution of 374:]

"At p. 189, Vol. VII, is found the probable error $x = \sqrt{[S(d_1^2) \div n]} \times \tan \frac{1}{2} \tan^{-1} cl = 707\sqrt{(S(d_1^2) \div n}$, where $S(d_1^2)$ is the sum of the squares of the errors, and , their number.

In the first case of 374, all possible errors, without regard to sign, are included between 0 and 0.5 of the last decimal place, their number is infinite, there is no greater density or accumulation of errors of one value between these limits than of another; therefore

$$\frac{S(d_1^2)}{n} = \int_0^{0.5} y^2 dy \div y = \tfrac{1}{3}y^2 + C = \tfrac{1}{12}.$$

Hence the probable error $x = \frac{1}{12}\sqrt{6} = 0.204$, instead of 0.25.

R. J. ADCOCK."

PROBLEMS.

375. *By W. B. Bates.*—A and B enter into partnership and gain \$200. Now six times A's accumulated stock (capital and profit) equals five times B's original stock, and six times B's profit exceeds A's original stock by \$200. Required the original stock of each.

376. *By Dr. H. Eggers.* — Divide a right angle into three parts, such that the tangents of the several angles are proportional to three given numbers.

377. *By W. E. Heal.*—If the equations,

$$x^2 + a\,x + b = 0$$
$$x^2 + a_1 x + b_1 = 0,$$

have a common root, find the remaining roots.

378. *By Isaac H. Turrell.*— O is the center of a circle circumscribing a triangle, and a, b, c, are the middle points of the sides opposite the angles A, B, C, respectively. If a circle be drawn through A to touch Ob, Oc, and another through B to touch Oa, Oc, prove that their common tangent passes through C.

379. *By Paul Peltier, A. M., Waterloo, Ill.*—If any number of circles touch one another in one point, all their polars which correspond to a common pole, pass through a single point.

380. *By Lieut. Chas. A. Stone, U. S. Naval Acad., Ann., Md.* — Find the equation of the curve in which the tangent of the angle which the tangent line makes with the axis of X, increases proportionally to the length of the curve.